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Computable Embeddings of Classes of Structures Under Enumeration and Turing Operators

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Abstract—In the paper we study the differences and partial characterizations of the Turing and enumeration computable embeddings of classes of structures

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1. INTRODUCTION

In the papers [1] and [2] the following two notions were introduced as a computability analog of Borel embedding.

Definition 1. Let K_0, K_1 be classes of structures in finite languages (for each class the language is the same).

1) We say that K_0 is *computably embeddable via an e-operator* into K_1 (and write $K_0 \leq_c K_1$) iff there are a function $f : K_0 \to K_1$ and an e-operator Φ such that $D(f(\mathcal{A})) = \Phi(D(\mathcal{A}))$ for any $\mathcal{A} \in K_0$ and and for any $\mathcal{A}_1, \mathcal{A}_2 \in K_0$

$$\mathcal{A}_0 \cong \mathcal{A}_1 \Leftrightarrow f(\mathcal{A}_0) \cong f(\mathcal{A}_1).$$

2) We say that K_0 is *computably embeddable via an Turing operator* into K_1 (and write $K_0 \leq_{tc} K_1$) iff there are a function $f : K_0 \to K_1$ and a Turing operator φ_e such that $\chi_{D(f(\mathcal{A}))} = \varphi_e^{D(\mathcal{A})}$ for any $\mathcal{A} \in K_0$ and and for any $\mathcal{A}_1, \mathcal{A}_2 \in K_0$

$$\mathcal{A}_0 \cong \mathcal{A}_1 \Leftrightarrow f(\mathcal{A}_0) \cong f(\mathcal{A}_1).$$

It follows from the next proposition then $K_0 \leq_c K_1$ implies $K_0 \leq_{tc} K_1$.

Proposition 2. $K_0 \leq_{tc} K_2$ *iff there are a function* $f : K_0 \to K_1$ *and an integer* $e \in \omega$ *such that* $D(f(\mathcal{A})) = W_e^{D(\mathcal{A})}$ *for any* $\mathcal{A} \in K_0$ *and for any* $\mathcal{A}_1, \mathcal{A}_2 \in K_0$

$$\mathcal{A}_0 \cong \mathcal{A}_1 \Leftrightarrow f(\mathcal{A}_0) \cong f(\mathcal{A}_1).$$

Proof. ($\Longrightarrow$) Obvious.

($\Longleftarrow$) Without loss of generality we can assume that $card(W_{e,s+1}^X - W_{e,s})^X \leq 1$ for all s and X. We denote via $T(a)$ the atomic sentence $a = a$ for each $a \in \omega$.

Suppose that $\mathcal{A} \in K_0$ is given. Define

$$S = \{s \in \omega : (\exists a)[T(a) \in W_{e,s+1}^{D(\mathcal{A})} - W_{e,s}^{D(\mathcal{A})}]\}$$

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